Experimental Study of Chaotic Vibrations in a Pin-Jointed Space Truss Structure

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In this paper, the dynamics of a pin-jointed space truss structure is studied. Experimental results demonstrated that the response of the truss, under sinusoidal excitation, exhibited broad-band, chaotic-like vibrations. It is believed that very small gaps in the joints create nonlinearities that lead to the chaos. When a tension cable was added to place the structure under compressive loads, the level of chaos was reduced. Numerical simulation of the truss dynamics including small gaps in the truss joints also showed similar chaotic behavior.

Introduction

LARGE practical space frame structures are often constructed from individual truss elements with pin joints. Also, in some planned space experiments deployable structures will be used.[1,2] In either case, small play or looseness in the joints represent nonlinear departures from ideal pin jointed structures or welded frame structures. The purpose of this study was to explore the effects of an actual pin jointed truss on the dynamic response of a multibay space truss structure.

It was our suspicion that small amounts of play in the joints could lead to chaotic dynamics in the response of the structure under periodic excitation. Chaotic dynamics in actual space structures might make it difficult to design active control system to damp out transient dynamics.

A second goal of this study was to see if internal stresses applied through a tension cable could reduce the pin-related nonlinearities and eliminate the chaotic dynamics.

The results of both studies lead us to believe that pin-connected trusses can behave quite chaotically under periodic forcing and that internal compressive loads may partially quench this chaotic behavior under low enough forcing. Numerical simulation for trusses having loose joints was also carried out to verify some of our experimental results.

The Structure

The structure which was tested in our laboratory is a 3.5 meter long 3-D truss with 16 bays, built with aluminum rods. The aluminum rods have a diameter of 0.476 cm (3/16 in) and have yield and ultimate stresses of 324 and 468 $\times 10^6$ N/m^2, respectively. The Young's modulus is 73.0×10^9 N/m^2 (10.6×10^6 psi), and the density is 2.77 gram/cm^3 (0.1 lb/in.^{3}). The structure is shown in Fig. 1, where the diagonal members on the front side were not drawn for the purpose of clarity. The cross section of the structure is triangular consisting of three equal-length rods of 22.2 cm (8.75 in.). The bay length is also 22.2 cm.

Figure 2 shows a typical joint of the truss. Six rod members were pinned to an aluminum plate. The pin and the hole both have diameters of 0.159 cm (1/16 in.). The rod members basically have two lengths: the shorter ones, 22.2 cm long in length with a through hole at each end, were used for longeron and pattern, and the longer ones, 31.4 cm long, also have a through hole at each end (the two holes have a 60-deg angle), were used for diagonal connections. When pinned together using the joint connectors, the rods form a three-dimensional truss structure.

Obviously, such a laboratory truss structure departs from the assumptions of an ideal truss, mainly due to the construction of joints. First, each joint has finite size; it can also have small rotations. Second, the rod members were connected to the joints with pins. Even though for each individual rod, one hardly feels any looseness of the pin connection, the whole structure has accumulated gaps and looseness. These two facts make the truss strongly nonlinear; the accumulated gaps and small rotations of the joints reduce the overall effective stiffness making the whole structure soft-spring-like. Therefore, we expect that the dynamics of the structure will not be regular, and that strange or chaotic motions may occur for periodic inputs. Chaotic vibrations occur in deterministic systems with strong nonlinearities. A number of nonlinear structural systems have been observed to exhibit chaotic vibrations under periodic excitation including buckled beams and beams with nonlinear boundary conditions (see Ref. 3 for an introduction and list of references to chaotic vibrations).

All parts of the structure were made of aluminum, and the total mass was approximately 2.3 kg. For the purpose of testing, two rubber bands were used to hang the truss on a ceiling. Recall that a free structure in space has six rigid body motions or modes: three translational and three rotational. All these modes are associated with zero natural frequencies. Hung with rubber bands, the first six natural frequencies of our truss had frequencies from 0.3–4.0 Hz, which is relatively small compared with the first experimental bending natural frequency of 44 Hz. Thus we refer to our truss as a free-free structure.

Finite Element Modeling

To have a better understanding of the truss structure and as a guide to later experiments, a finite element program was developed for simulating the natural frequencies and mode shapes of the system with zero gaps in the joints.

In our analysis, each rod element was modeled as a spring of stiffness EA/L with concentrated mass of $m/2$ at each end; where E is the Young's modulus, A the cross-sectional area, L the length of the rod, and m the mass per unit length. For an

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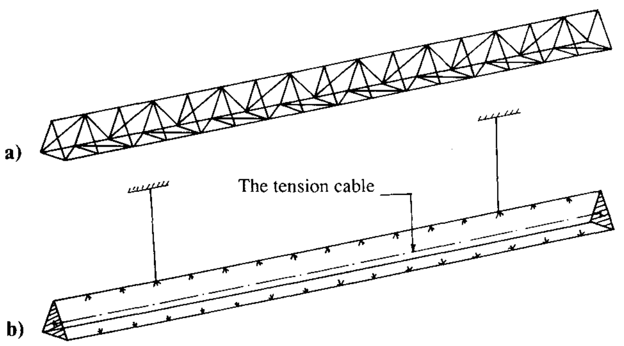

Fig. 1 a) The experimental truss structure; b) the cable tensioned truss hung from the ceiling by two soft rubberbands.

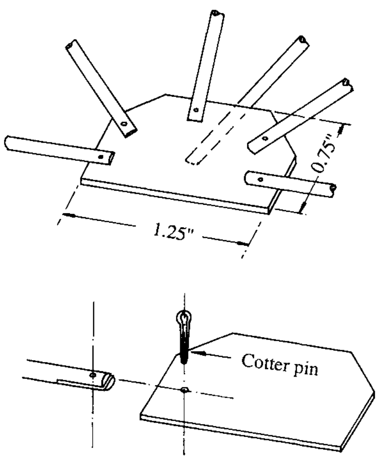

Fig. 2 Sketch of a typical nonlinear joint.

ideal truss, the accuracy of this algorithm is good for the first few lower modes. Since this lumped mass approximation is equivalent to adding some constraints to the structure, the numerical natural frequencies are expected to be slightly higher than the real ones.

Our 16-bay truss had 51 nodes and 147 members. Therefore the global mass and stiffness matrices have dimensions of 153×153 ($51 \times 3 = 153$). The first and the second bending modes had calculated natural frequencies of 65.5 and 167.7 Hz, respectively, and had mode shapes shown in (a) and (b) of Fig. 3. The first calculated twisting mode had a natural frequency of 98.8 Hz and has a mode shape as shown in (c) of Fig. 3, as expected.

Note that the above numerical results are for an ideal pin-jointed truss structure. However, the laboratory truss had nonideal joints. In our experiments, the joint was not an infinitesimal point but had a finite-sized small plate, to which six members were attached. In the following section, the results of modal testing are summarized and compared with the numerical results in this section.

Experimental Modal Shapes and Frequencies

Experimental modal surveys of the truss structure were carried out with the use of a Zonic Modal Analysis System (6088), which has the capabilities of measuring frequency response functions (FRF) and performing modal analysis. The modal survey procedure was divided into two parts: data measurement and data processing. In the first step, 18 ($= 6 \times 3$) nodes in the truss were chosen to represent the coordinate points for obtaining FRF's. A small volume accelerometer having a mass of 1 g was used to measure the response at a node in the middle

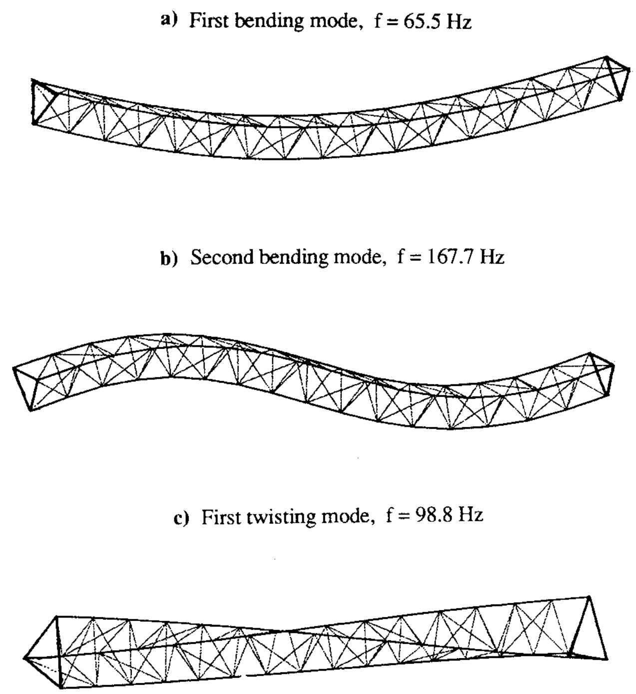

Fig. 3 First few natural frequencies and the corresponding mode shapes for an ideal linear truss.

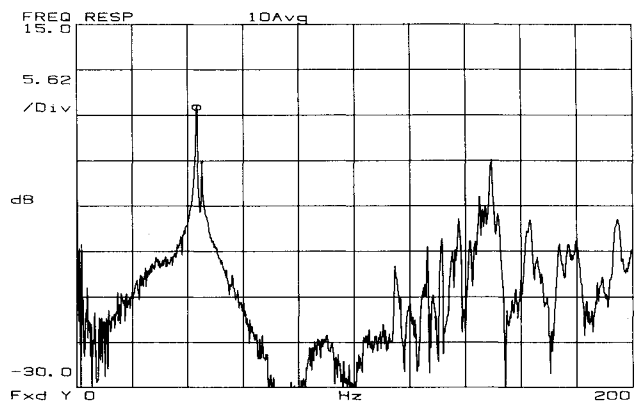

Fig. 4 A frequency response function of the truss excited by an impulse input.

of the truss (along the longeron direction). A highly sensitive hammer was used to hit the truss at the chosen nodes in the vertical direction generating impulse inputs. Both signals were sent to the Zonic electronics to be processed. A typical frequency response curve is shown in Fig. 4. We note that there are two close peaks in the FRF curve which correspond to the first bending mode. For an ideal long triangular truss, which possesses a symmetry in the plane of the cross section, each bending mode has two identical natural frequencies due to the symmetry. For our truss, this symmetry does exist, but only approximately. This is the reason for the splitting of the two peaks. We point out that a single, main-peak, frequency-response curve was also observed. In the modal analysis processing, only the main peak, with a bigger circle on the top, was used for the first bending mode.*

*Note that there are many small peaks in the higher frequency range in Fig. 4. For a welded-joint truss, its frequency response curve is virtually smooth in a similar frequency range. Thus we believe that the small high-frequency peaks are caused by the nonideal joint.

In the second step, the FRF's were transferred into a modal analysis package in the instrument for further processing. This included extracting the modal-frequencies, modal-shapes, damping coefficient for each mode from the FRF data. It should be pointed out that the software in most modal analysis packages assumes that the structure is linear. Thus, the pictures of mode shapes in Fig. 5 must be considered as for an "equivalent linear" structure. For a nonlinear structure the idea of mode shapes is not well defined.

Figure 5 shows the first two lowest bending mode shapes with natural frequencies of 44.38 and 106.65 Hz, respectively. Unlike the mode shapes in the numerical simulations (see Fig. 3), the experimental mode shapes look distorted, which indicates strong nonlinearities in the truss joints. In fact, in another experiment of a truss with welded joints, the mode shapes of the bending modes looked much more classical than the ones in Fig. 5. The two experimental natural frequencies are also well below the values of their numerical counterparts. This is partially due to the modeling process in the numerical simulation,† and mainly due to the nonlinear joints: the large amount of accumulative gaps as well as small rotations render the truss "softer" elastically than the linear theoretical model.

Another impact of the nonlinear joints is that the natural frequencies measured from the peaks in the FRF curves (see Fig. 4) were rarely the same in different measurements. The first bending frequency in Fig. 4 is 43.25 Hz; however, the one in Fig. 5 is 44.38 Hz, which was obtained in an averaged sense. In most cases, the first bending frequencies varied in the ranges of 42-45 Hz in experiments.

Chaotic Oscillations of the Structure

The forced dynamic response of the truss was studied by applying a sinusoidal force and measuring the output signals with the use of the accelerometer. The accelerometer was mounted close to a joint at one of the two ends. The driving frequency was in the range of $25 \sim 75$ Hz, which was in the neighborhood of the first "natural" frequency of 43 Hz.

Unlike an ideal linear truss, which has a periodic output when forced by a periodic input, the response of the truss was often random-like for relatively small forcing. This strangeness of the response was indicated in the broadband power spectrum of the output signal as shown in (b) of Fig. 6. If the forcing amplitude was sufficiently small, the broadband power spectrum of the response would disappear, and many clear spikes (peaks) would emerge indicating different mode frequencies and their combinations. Part (a) of Fig. 6 shows the power spectrum for the forcing signal (which was directly measured from the shaker with the use of an accelerometer) which had one principal frequency and several superharmonic components.

In this study, we define a broadband response of a system under periodic forcing to be chaotic. This criterion is for experimental convenience. Mathematically one should show that the largest Lyapunov exponent for the vibration output is positive in order to label the motion chaotic (see Chapter 5 of Ref. 3). However, experimental measurement by Lyapunov exponents has not been reliably developed. Thus, we rely on the fast Fourier transformations (FFT) signal to define chaos. The appearance of chaotic dynamics of the truss once again suggests that the nonlinearities associated with the pin-truss joints complicate the dynamics of the system.

Connections of the Truss with the Shaker

In the dynamical testing, we found that when the structural response become chaotic (with broadband spectrum), the chaos was also fed back to the shaker. The signal from the armature of the shaker, measured by another accelerometer, was contaminated; the spectrum of this signal also contained many narrow-banded peaks. In our experiments, the truss was originally connected to the shaker with a rigid steel rod. Obviously, when the motion of the truss was chaotic, the motion of the armature became chaotic too.

To study the dynamics of the truss alone, we designed a decoupling device to isolate the chaos of the truss. The objective of such a device was to supply a force to the truss at the same time separating the motion of the truss from that of the shaker. Some mechanical properties of the truss and the shaker are presented in the following to help understand the nature of coupling. The overall mass of the truss is approximately 2.3 kg, and its first experimental natural frequency is about 44 Hz. The armature mass is 1.9 kg, and the associated stiffness is 21.45 kg/cm. The damping coefficient of the shaker is critical, i.e., its damping ratio is equal to one. A schematic diagram of the vibration system is shown in Fig. 7.

We designed a spring with stiffness k to connect the truss with the shaker. In our laboratory tests, a spring with stiffness of 17 kg/cm was used to connect the truss with the shaker. It was observed that adding the spring connector was an effective way to get rid of chaos in the motion of the shaker. For a more accurate analysis, the truss and the shaker have to be considered to be a single dynamical system with an electrical sinusoidal forcing input. In this paper, we only consider the dynamics of the truss by assuming that the shaker can provide approximately a harmonic input to the truss.

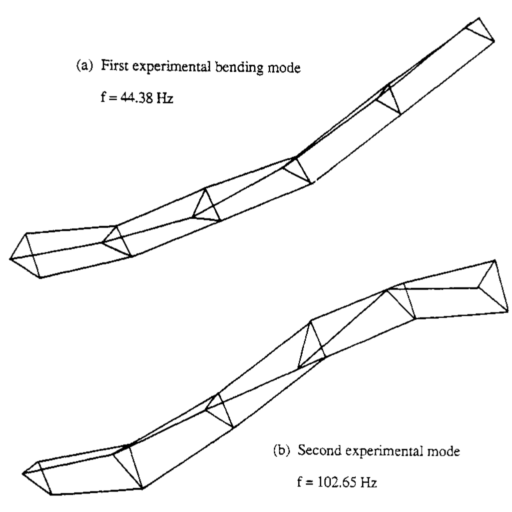

Fig. 5 Experimental natural frequencies and their mode shapes.

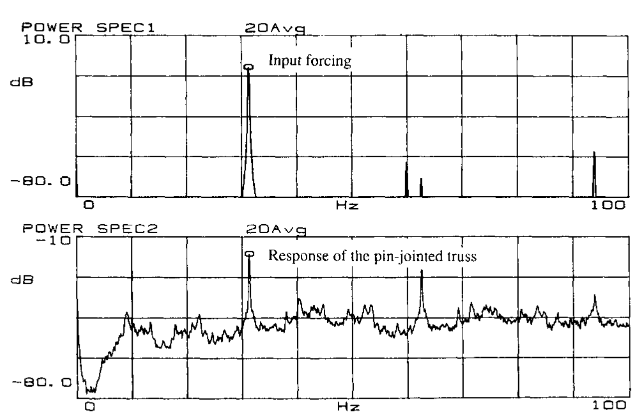

Fig. 6 Typical power spectra for the input and output signals (note that the power spectrum for the system response under periodic input is broadband).

†For a similar truss structure with welded joints, the first natural frequency from simulation is approximately 4% higher than the value from experimental testing. This error becomes bigger for the higher modes.

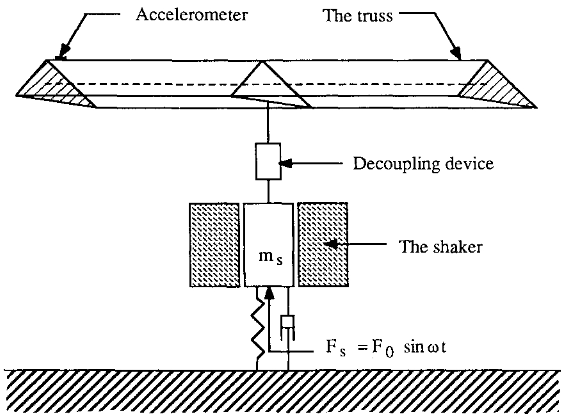

Fig. 7 The experimental setup for the dynamic tests.

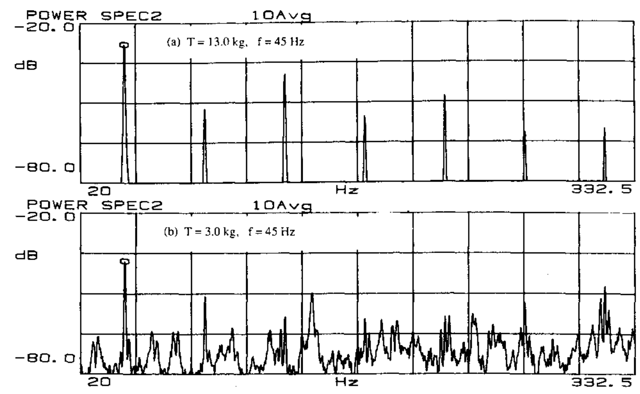

Fig. 8 Power spectra demonstrating the effects of cable tensions.

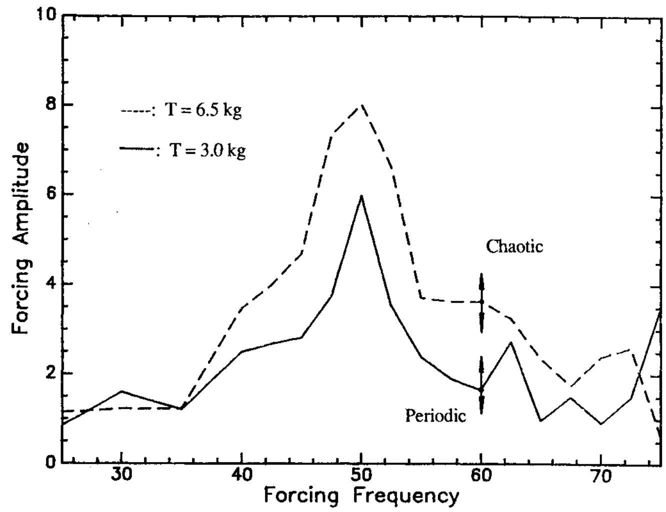

Fig. 9 Criteria for chaos in the forcing frequency and amplitude plane.

Effects of Cable Tensions

We have seen that the dynamics of the truss was complicated because of the nonlinear pin joints. To control the dynamical response, a steel cable was added to the truss in the direction along the longerons in hopes of bringing back the dynamics of the truss into the linear regime at least to reduce the level of chaos.

To this end, two triangular thin aluminum plates (0.3175 cm thick) were attached to each end of the truss (see Fig. 7). Then a tension cable was connected to the two centers of the two plates. The total length of the cable can be adjusted by some special screw nuts. A full wheatstone bridge was designed to measure the tension forces in the cable. A schematic configuration after the addition of the plates and the cable is also shown in Fig. 1.

The first natural frequency of the steel cable is given by the formula

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where $L \approx 3.556$ m is the length of the cable, $\rho = 7.33 \times 10^{-3}$ kg/m is the mass density, and T is the tension it carries. The typical load of the cable will be in the range of $0 \sim 25$ kg, which corresponds to the fundamental frequencies of the cable in $0 \sim 25$ Hz. It is seen that these frequencies are below the natural frequencies of the truss. In experiments, the interactions between the truss and the cable were weak, and the cable tension was considered to be constant in each experiment.

Figure 8 demonstrates the power spectra for the responses of the truss for different cable tensions. In Figure 8, the driving frequency was 45 Hz. When the cable tension was set to be 13 kg, we observed a periodic output of the truss for certain input forcing. The output signal only contained a few clear peaks in the frequency domain (see (a) in Fig. 8). To see the effects of the cable tension, we gradually reduced the tension to 3 kg and waited for some time to let the vibration settle down, the spectrum became broadbanded as shown in (b) of Fig. 8. Generally the more the T was reduced, the more broad the power spectrum was. In some cases, the level of the power spectrum was also increased even for fixed input forcing level.

As was pointed out, the joints of the truss have small gaps and can rotate slightly when external forcing was added. The results in Fig. 8 tell us that when the cable was tightened, some gaps were closed. The small rotation of some joints was also prevented. Thus one expects that under compression the truss behaves close to its linear regime and that the degree of chaos, if any, is reduced. For some forcing frequencies, strong chaotic motions could not be deleted by tightening the cable. The truss is three dimensional, and the joints connect individual rod members in many different directions. One cannot expect to get rid of all loose gaps with a single cable in the longeron direction.

Inspired by the results in Fig. 8, we tried to find criteria for chaos, in the frequency domain, for different cable tensions. In general, the power spectrum of an output signal contained many peaks such as the one in (a) of Fig. 8 before the onset of chaos. For each fixed frequency, we gradually increased the driving amplitude until the spectrum of the response of the truss became broadbanded such as the one in (b) of Fig. 8. When this transition occurred, we recorded the forcing amplitude. Repeating the above procedure for different driving frequencies and different cable tensions, we obtained chaos criteria for the truss, as shown in Fig. 9. The horizontal axis represents the driving frequency, and the vertical axis is the acceleration of the shaker, which is proportional to the driving force applied to the truss. It is clear in the figure that when the cable tension is increased, the forcing amplitude is also increased for the onset of chaotic responses.

These results hold out the promise that the nonlinear effects of pin joints or deployable truss elements may be reduced if initial compression in the truss is applied with tension cables.

Dynamic Analysis and Some Numerical Results

As we have seen in the previous sections, the forced response of the truss, having joints with loose gaps, was random-like. In this section we try to model the structure more closely by including the small gaps in each joint and perform numerical simulations. The question that we want to answer is that if the random-like response in experiments is an intrinsic feature of the loose joints in the truss structure. Another possible joint nonlinearity is dry friction, but our first model will neglect this effect and assume only gap or play-type nonlinearities.

Recall that each member was modeled as a linear spring previously, with two masses at the two ends, without considering the effects of the gaps in the joints. The results of the modal shapes and the associated natural frequencies were for this idealized case. Referring to Fig. 10, however, for each member, there were small clearance or looseness in the joints of each member due to the machining error of construction. The diameter of the holes might be slightly bigger than the diameter of the pins, causing looseness. For each single member, this looseness is hardly felt, but as a whole of the structure, the looseness becomes obvious. Another geometric-elastic nonlinearity in pin-type joints is due to Hertz contact between the pin and the plate hole. This effect was not considered to be the principal cause of the chaos, however.

The looseness in the two joints for each member can be conveniently included by a single gap in the spring-mass model,[5] as shown in (b) of Fig. 10. As usual, this spring has stiffness of EA/L and two masses; each has mass of $m/2$ ($m = AL\rho$) at the two ends plus a gap with a free-play of $2d$. The force-displacement relation is shown in (c) of Fig. 10. When the displacement u ($=x^2-x^1$) is bigger than d or smaller than $-d$, the force is proportional to the increase in u. When $|u| \le d$, however, the spring does not experience any strain and, thus the force is zero. This situation is a typical trilinear spring model, and the stiffness has a mathematical description[6]

$$F(u) = ku - \frac{k}{2}\left\{|u+d| - |u-d|\right\} \tag{1}$$

Once again using the finite element assembling technique and considering the looseness in the joints, one can derive the governing equation of the motion in matrix form:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(x)\} + \{f\}\sin\Omega t \tag{2}$$

where $\{x\}$ is a vector function representing the nodal displacments of the truss; $[M]$, $[C]$, and $[K]$ are the usual mass, damping, and stiffness matrices, $\{F(x)\}$ is a residual force vector due to the loose joints, and $\{f\}\sin\Omega t$ is a sinusoidal forcing term. For our truss structure, where there are 147 members and 51 nodes, the number of degrees of freedom is 153; it means that the previous matrix equation includes 153 second-order coupled equations, which is fairly big for a moderate sized computer if any numerical solutions are attempted.

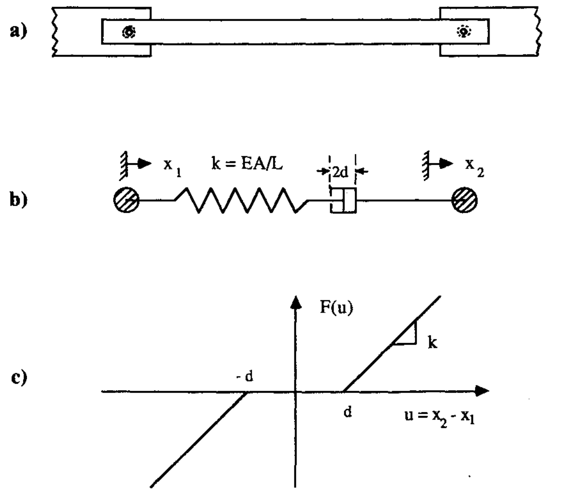

Fig. 10 a) A pin-jointed rod member in the truss; b) an idealized model; c) the force-displacement curve.

Because of the complexity in $\{F(x)\}$, no analytical solution is available.

For the purpose of reducing the degrees of freedom of the governing equations, we seek a modal matrix $[P]$, which satisfies $\Omega^2[M][P] = [K][P]$ for the system (2). The matrix $[P]$ can be normalized such that $[P]^T[M][P] = [I]$ and $[P]^T[K][P] = [\Omega^2]$; where $[I]$ is an identity matrix, and $[\Omega^2]$ is a diagonal matrix consisting of natural frequencies squared in the order from lower to higher. Making a transformation from the physical displacements $\{x\}$ to the modal displacements $\{y\}$ by $\{x\} = [P]\{y\}$, and replacing $\Omega_1 t$ by τ, where Ω_1 is the first natural frequency of the truss, we have

$$\{\ddot{y}\} + [C^1]\{\dot{y}\} + [\omega^2]\{y\} = \{F^1(y)\} + \{f^1\}\sin\omega\tau \tag{3}$$

where

$$\{F^1\} = [P]^T\{F(P\{y\})\} \text{ and } \{f^1\} = [P]^T\{f\}$$

The above matrix equation also contains 153 second-order differential equations. But based on numerical simulation results, we observed that when the driving frequency ω is close to the fundamental natural frequency, the magnitudes of the response of the first few modes are much bigger than the magnitudes of the higher modes. Practically, the higher modes can be neglected to obtain approximate solutions. How many higher modes can be neglected, or equivalently how many lower modes should be retained, depends on the specific system. Details of the procedures of the numerical implementa-

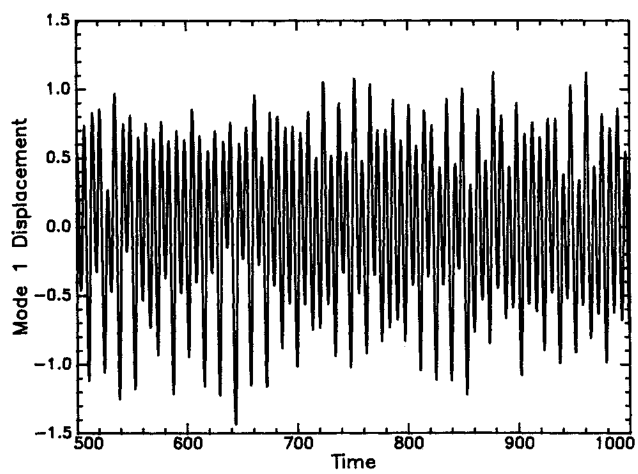

Fig. 11a Numerically calculated time responses for the first bending mode of the truss with and without gaps, gap = 0.1, damping coefficient = 0.1, forcing frequency = 59 Hz, forcing amplitude = 0.1.

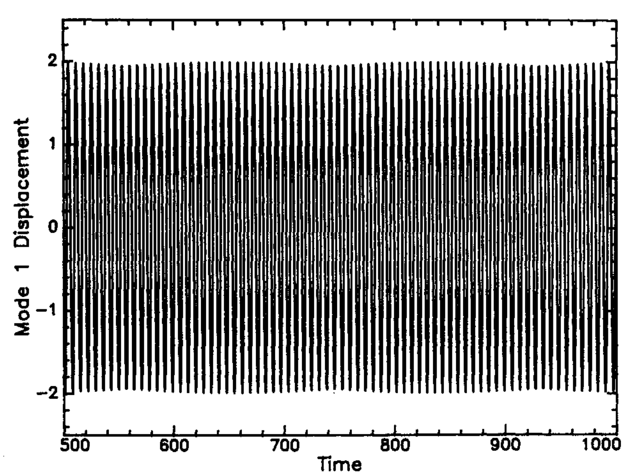

Fig. 11b Gap = 0.0, damping coefficient = 0.1, forcing frequency = 59, forcing amplitude = 0.1.

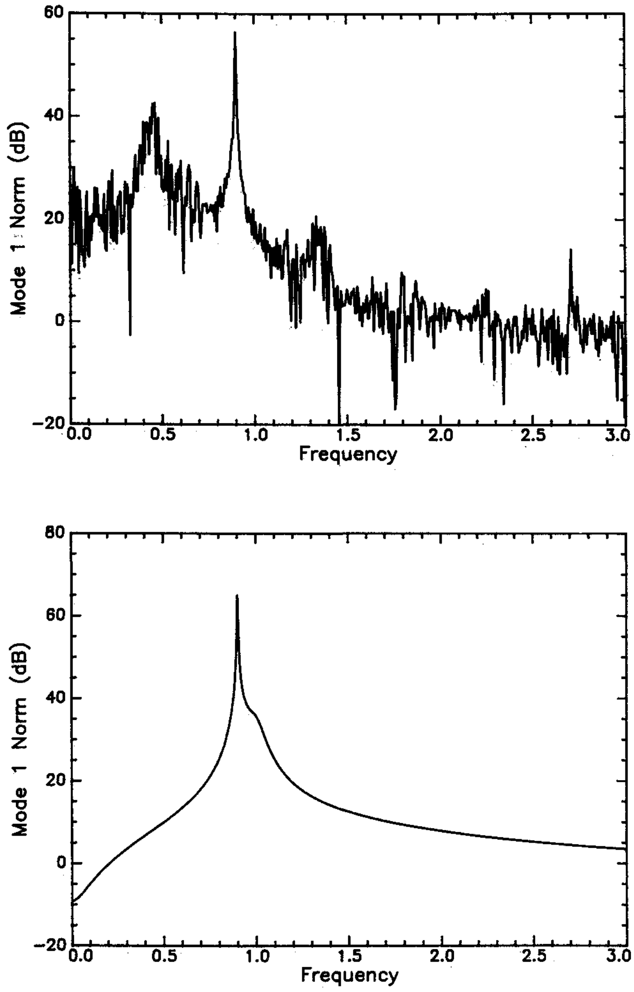

Fig. 12 Frequency responses of the time signals in Fig. 11.

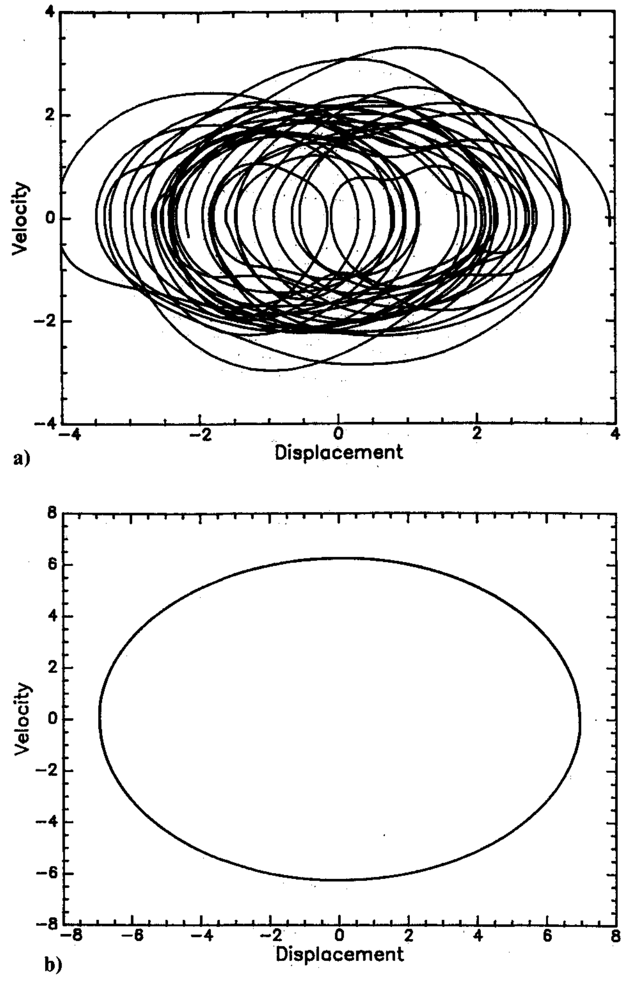

Fig. 13 Numerically calculated phase trajectories of the vertical motion of the truss in a time interval of 500 ~ 700 nondimensional time units for a) the gapped truss, the trajectory is chaotic under a sinusoidal forcing; b) the idealized truss (no gaps), the trajectory is periodic at the same input parameter values.

tion for our system will be described in our next report (for other numerical techniques, see Ref. 7). Here we only present some preliminary results. In our simulations, the first five modes were chosen. The first two correspond to the first bending modes having the natural frequency of 65.5 Hz for the idealized truss, the third one corresponds to the first twisting mode having the natural frequency of 97.8 Hz, and the fourth and fifth ones correspond to the second bending modes having the natural frequency of 167.7 Hz.

In the following simulations, we assumed that the damping coefficients in the matrix $[C^1]$ would be 0.1, initial conditions would be 0. The forcing frequency and amplitude were 59 Hz and 0.1 respectively. Figure 11 shows the time responses of mode 1 of the truss in cases with and without gaps in the joints. The input was sinusoidal and was applied to the truss at two nodes in the middle along the vertical direction (z-axis). A direct look at these two time responses immediately suggests that the dynamical behavior of the gapped structure appears strange where no periodicity can be observed. While for the response of the linear structure the periodicity of the response is quite obvious. Notice that the transient responses were deleted to eliminate the effect of initial conditions in both cases.

As in the experimental case, we once again used the power spectrum technique to discriminate the dynamical behavior for the numerical responses. Figures 12a and 12b show the FFT for the time responses in Fig. 11. On the one hand, a smooth looking curve with a single peak was obtained for the periodic response of the linear truss. On the other hand, the FFT curve in Fig. 12 for the gapped truss response is rough and broadband. The main peak in Fig. 12 corresponds to the forcing frequency (nondimensionalized).

To have a better understanding in the behavior of the truss again in both cases, we also simulated trajectories in phase space for a node at one end of the truss, as shown in Fig. 13. Figure 13a shows the phase trajectory for the gapped structure, which is random-like; while Fig. 13b shows the phase trajectory for the linear structure, which is clearly periodic. The parameter values used here were the same as in the previous case. The two phase trajectories were in a time period of 500 ~ 700 nondimensional time units.

Conclusions and Future Work

The experimental study on the pin-jointed truss has shown that the dynamics of the structure could be extremely complicated by the nonlinear joints. The modal frequencies of the truss were considerably lower than the ones for the linear truss. The dynamic response was clearly chaotic as indicated by its broadband spectra. By adding a tension cable along the longeron direction of the truss, the degree of chaos was lowered.

Numerical simulations for our 16-bay truss were also carried out by including the small gaps of the joints in our mathematical model. The preliminary simulation results indeed tell us that the dynamics of the truss is complicated by the looseness of the joints. More extensive and complete simulations are still needed to fully understand the system dynamics.

Some theoretical analyses for interpreting the experimental results have been reported. A bilinear, periodically forced, one-degree-of-freedom oscillator has been shown to yield chaotic motions via a period-bifurcation sequence.[8] Another system, which also yields chaotic motions when the feedback is strong enough, is a trilinear oscillator.[9] A preliminary study for a single degree of freedom oscillator having trilinear stiffness, as shown in (c) of Fig. 10, is being carried out, and period-doubling bifurcation sequence and thus chaos has been observed.[10] The result can be employed to understand the dynamics of our laboratory truss.

This study clearly shows that the practical construction of truss-type structures in space may bring in nonlinearities in the properties of the joints. The observation of chaotic behavior in such structures suggests that linear methods of theoretical and experimental analysis may not be useful and that the new tools of nonlinear vibrations may be needed to understand the dynamics of real structures in space.

Acknowledgment

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